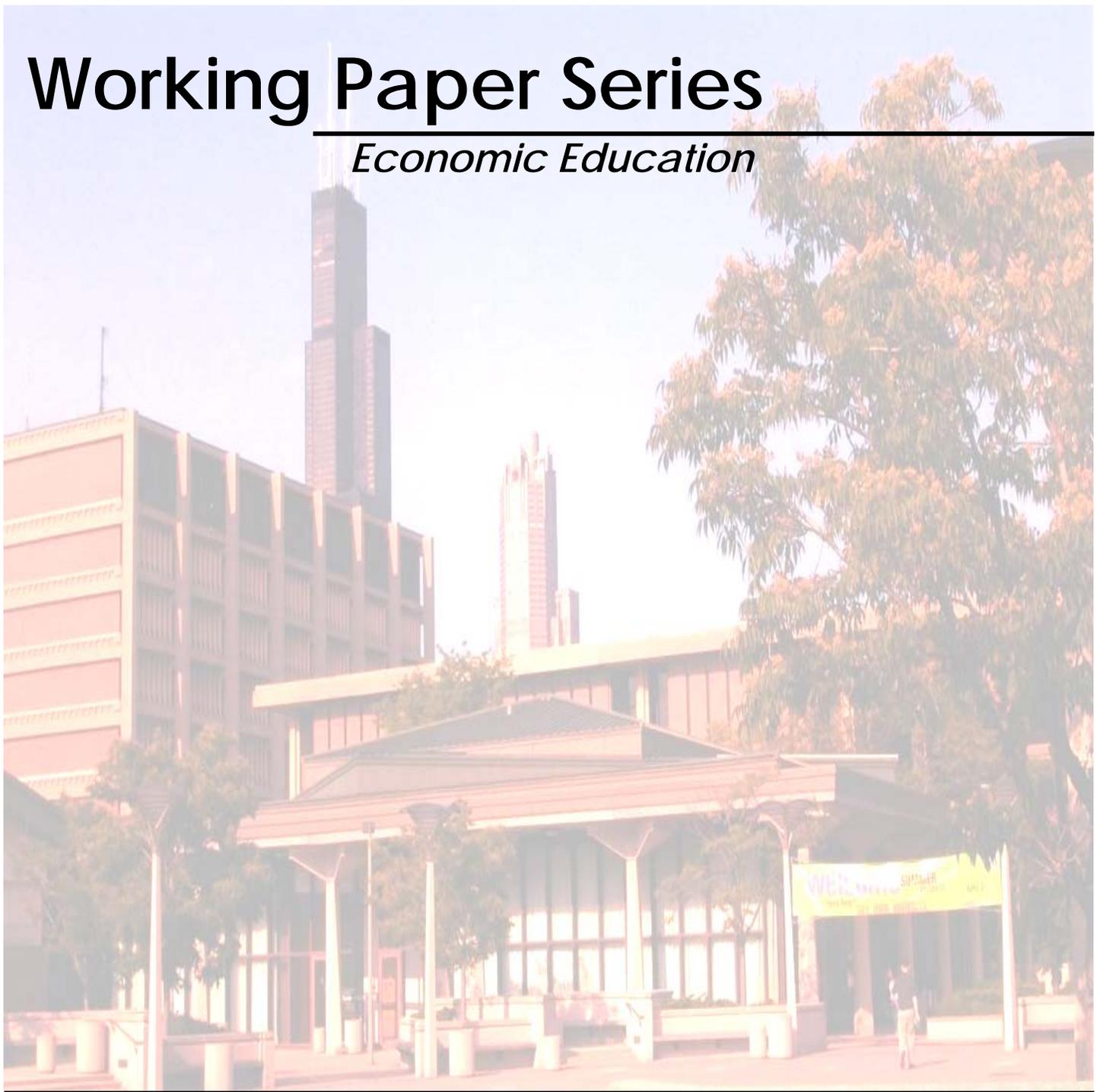


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## *Teaching the Law of Downward Sloping Demand and Irrational Behavior*

Richard M. Peck, Associate Professor  
University of Illinois at Chicago

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**Center for  
Economic  
Education**

Department of Economics (m/c144)  
University of Illinois at Chicago  
601 South Morgan Street  
Chicago, Illinois 60607  
Tel 312-996-2684  
Fax 312-996-3344  
<http://cee.econ.uic.edu>

# Teaching the Law of Downward Sloping Demand and the Becker Irrationality Argument

Richard M. Peck  
UIC Economics Department  
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## **Introduction:**

In Gary Becker's well known 1962 paper "Irrational Behavior and Economic Theory" Becker demonstrates that rationality is not necessary to generate downward sloping compensated demand curves. Becker considers two notions of "irrationality". The first is irrationality as inertia – consumers try to do what was done previously and are insensitive to price changes. The second is irrationality as random behavior. His key conclusion is that rationality is not necessary to generate downward sloping market demand curves.

In this paper we focus on irrationality as random behavior. We show how to parameterize the process of randomly choosing a bundle on the budget set. This allows the random selection of bundles on the budget line to be simulated using the random number generator of widely available spread-sheet programs such as Excel. Students can vividly see that while some individuals do not exhibit downward sloping demand, the market demand curve does slope down. Even more striking is the demonstration that market income compensated demand curves slope down as well. Put more technically, while individual behavior may not be rational, average behavior conforms to the weak axiom of revealed preference in a two good framework, provided the number of participants is large. This means that on average, rationality prevails.

The parameterization of the random process provides its own set of insights. The

parameterization maps points on the unit interval to points on the budget line. This parameterization indicates that the average or mean behavior that arises from random choice on the unit interval always corresponds to a Cobb-Douglas utility function. This immediately implies that uncompensated and income compensated demand curves derived from average market behavior have to slope down, when the sample size is big enough. There are no restrictions on probability distribution over the unit interval except the requirement that a mean for the distribution exists. Put differently, the case considered by Becker, uniform distributions over the budget line corresponds to the uniform distribution on the unit interval. But this is only of many probability distributions that will generate market behavior consistent with rationality.

The other more diffuse point that emerges from this exercise is a schematic depiction of how most economists, particularly applied economists, use notions of rationality in interpreting real world phenomena. . Economists are not all that interested in explaining all the behavior recorded in a data set – they are interested in explaining average behavior or its equivalent, market behavior. As long as average behavior is rational, rationality is a convenient device for summarizing and explaining average behavior. Moreover, the rationality framework is very useful for making predictions about market behavior and average behavior. It suggests hypothesis about the signs and magnitudes of estimated coefficients.

An important distinction between economists and psychologists is that economists are primarily interested in the average, or market, behavior of relatively large populations. For example, demand elasticity estimates from large data sets really tell us about average population behavior. Most questions of policy design and design usually focuses on average behavior – a ten percent increase in cigarettes prices causes quantity demanded to fall by six percent. Psychologists

are more interested in deviations from average – after all, clinical applications of psychological insights are almost exclusively focused on individual pathology.

**Parameterizing Random Choice.**

Suppose choices are made randomly – the individual throws a dart at his budget line.

To formalize this suppose  $\alpha$  is a random variable on the interval  $[0,1]$  with density function  $f$ , that is for  $\alpha \in [0,1]$ ,  $f(\alpha) \geq 0$  while for  $\alpha \notin [0,1]$ ,  $f(\alpha) = 0$ . Since  $f$  is a density function

$$\int_0^1 f(\alpha)d\alpha = \int_{-\infty}^{\infty} f(\alpha)d\alpha = 1$$

Note given a random variable  $z$  with support  $(-\infty, \infty)$  it is possible to generate a random variable  $\alpha$  with support  $[0,1]$  by letting

$$\alpha = \frac{z^2}{1+z^2}$$

that maps  $(-\infty, \infty)$  into  $[0,1]$ . The probability density for  $z$  along with this definition of  $\alpha$  will induce an appropriate density  $f$  for  $\alpha$  on  $[0,1]$ .

For a given  $t$ , suppose that  $x$  is given by:

$$x = (1 - \alpha) \frac{I}{p_x}$$

while

$$y = \alpha \frac{I}{p_y}$$

This is basically a parametric form of the budget line that maps the random variable  $\alpha$ , into the budget line. Note, that for any  $\alpha$  between zero and 1, we have, multiplying the first equation by  $p_x$

$$p_x x = (1 - \alpha)I$$

And multiplying the second equation by  $p_y$  :

$$p_y x = \alpha I$$

Now adding the two resulting equations gives:

$$p_x x + p_y y = (1 - \alpha)I + \alpha I = I$$

If the expected value of  $\alpha$  exists is  $\bar{\alpha}$ , the average values of  $x$  and  $y$  are given by

$$\bar{x} = (1 - \bar{\alpha}) \frac{I}{p_x}$$

And

$$\bar{y} = \bar{\alpha} \frac{I}{p_y}$$

These of course are demand functions generated by Cobb-Douglas utility functions

$$U(x, y) = x^{\bar{\alpha}} y^{1-\bar{\alpha}}$$

Since average demands for  $x$  and  $y$  are generated by a utility function, compensated demand curves will slope downward. From the function form of the average demand curves, it also immediately clear that average uncompensated demand is also downward sloping. As long as the distribution of  $\alpha$  is stable, independent of prices and income and the mean of  $\alpha$  exists, mean demand will be well behaved – downward sloping and the weak axiom of revealed preference will be satisfied by average behavior. Not all economists agree that this process of randomization captures “irrationality” – but it certainly doesn’t correspond to maximizing a stable utility function subject to a budget constraint.

If  $\alpha$  is uniformly distributed on  $[0, 1]$ , then this construction means that there is a uniform probability distribution on the budget line – choice corresponds to throwing a dart at the budget

line to determine consumption of x and y. Here,  $\bar{\alpha}$  is equal to 1/2, and average demand for x and y are

$$\bar{x} = \frac{I}{2p_x}$$

And

$$\bar{y} = \frac{I}{2p_y}$$

These values of x and y correspond to the midpoint of the budget line. This is the case considered by Becker.

### **Simulations:**

We can make this more concrete by actually generating an example. Here we have used Excel's random number generator to simulate the process of throwing darts at the budget line. One can think of this as corresponding to individuals just randomly selecting points on the budget line. Here the underlying probability distribution of  $\alpha$  is uniform, so that the distribution of choices on the budget is also uniform. This means that the mean amount consumption of x and y will be  $I/(2p_x)$  and  $I/(2p_y)$  respectively. Here we have assumed that income is 100 and the price of y is one. We are varying the price of x.

Each cell is interpreted as an observation for one single individual who is choosing randomly along the budget line. All individual have the same budget line.

## Example

|    | Income     | 100      | 100      | 100      |          |  |
|----|------------|----------|----------|----------|----------|--|
|    | Price of X | 2        | 1        | 4        |          |  |
| 1  | 42.19723   | 22.89949 | 14.7727  | -19.2977 | 27.42453 |  |
| 2  | 38.20636   | 51.89661 | 17.62564 | 13.69025 | 20.58073 |  |
| 3  | 13.32536   | 97.72136 | 14.44646 | 84.396   | -1.1211  |  |
| 4  | 17.93568   | 42.59624 | 3.458134 | 24.66056 | 14.47755 |  |
| 5  | 35.23833   | 58.82239 | 18.11531 | 23.58405 | 17.12302 |  |
| 6  | 31.46632   | 15.78467 | 20.62559 | -15.6817 | 10.84073 |  |
| 7  | 38.40506   | 42.32787 | 18.33532 | 3.922812 | 20.06974 |  |
| 8  | 2.858528   | 23.17582 | 17.08687 | 20.31729 | -14.2283 |  |
| 9  | 30.9024    | 30.82318 | 1.504554 | -0.07922 | 29.39784 |  |
| 10 | 13.01689   | 35.57998 | 24.72265 | 22.56309 | -11.7058 |  |
| 11 | 12.92227   | 52.10248 | 10.0884  | 39.18021 | 2.833867 |  |
| 12 | 16.97074   | 19.56969 | 15.10505 | 2.598951 | 1.865688 |  |
| 13 | 4.00333    | 15.07884 | 13.63484 | 11.07551 | -9.63151 |  |
| 14 | 26.09929   | 10.44348 | 15.84604 | -15.6558 | 10.25325 |  |

Initially the price of x is 2 and then it drops to 1, so that the budget set swings out. Some individuals appear to respond to this price drop by decreasing the amount of x purchased. For example, individual number 1 cuts her consumption in half after the price drops. Indeed 4 out of the 14 individuals (different people) exhibit upward sloping demand – that is, 29 percent for each pair-wise price comparison. In all, 8 of the 14 individuals, well over half, exhibit an upward sloping demand curve (x as a Giffen-good) at least once.

Here are total and average amount purchased of x: a clear downward sloping demand curve is exhibited.

|   |          |          |
|---|----------|----------|
| 2 | 977.4612 | 22.73166 |
| 1 | 1830.647 | 42.57319 |
| 4 | 625.3926 | 14.54401 |

On average, consumption behavior exhibits downward sloping demand. As Becker stresses this reflects scarcity and the fact that a higher price indicates greater scarcity – the budget line shrinks as price rises. Thus downward sloping market demand will arise, even if individual choice is random.

**Econometrics**

If regress average amount purchased on price, that is, estimate for this data

$$Q_x = a - bP_x$$

we find that

$$a = 4.9$$

and

$$-b = -8.6$$

|              | <i>Standard</i>     |              |               |
|--------------|---------------------|--------------|---------------|
|              | <i>Coefficients</i> | <i>Error</i> | <i>t Stat</i> |
| Intercept    | 46.66701            | 10.3093      | 4.526691      |
| X Variable 1 | -8.59317            | 3.896549     | -2.20533      |

If we regress using individual observations, we get:

|              | <i>Standard</i>     |              |               |
|--------------|---------------------|--------------|---------------|
|              | <i>Coefficients</i> | <i>Error</i> | <i>t Stat</i> |
| Intercept    | 46.66701            | 3.871664     | 12.05348      |
| X Variable 1 | -8.59317            | 1.463351     | -5.87225      |

The fact that the coefficient are identical comes from the fact that regression coefficient are determined so that the regression line goes through sample means.

There is another pedagogical point about econometrics and sample size. Using the type of Monte Carlo simulation described, demonstrating that when the price change is small, the sample size typically needs to be large to capture the effect is straight forward. This is a basic point: When the sample size is small and variations are not large, it may not be possible correctly to discern underlying relationships.

Finally this exercise bring out an important broader point – economics, particularly empirical economics is typical focused on behavior at the mean, that is, average behavior. The basic ideas of economics are very useful for explaining market behavior, and the notion of rationality is typically very useful for understanding and predicting average behavior. This emphasis on averages is reinforced by the statistical tools that economists use: Regressions are really about average behavior. Thus the fact that a small or moderate number of individuals in the sample behave in a manner inconsistent with downward sloping behavior is not particularly upsetting to the modal applied econometrician.

## Compensated Demand Curves

In some sense what we have described above isn't a very strong test of rationality because with any two observations, each drawn by random choice from non-intersecting budget lines, are always consistent with maximizing some stable utility function. Of course, with more observations on the same individual, holding price and income constant, choices will be different, thereby revealing the random aspect of choice process. But it is possible to falsify the utility maximizing hypothesis with only two observations. In particular, examining choices with income compensated changes in price is a tighter test of rationality since only two observations may exhibit behavior inconsistent with utility maximization.

Recall the definition of an income compensated price changes: The price of x changes but income is also adjusted so that the individual can still just purchase her previous bundle at the new prices. To see what is involved in more detail, suppose income I and  $p_y$  equals one and the price of x changes from  $p_x$  to  $p'_x$ . If  $\bar{x}$  and  $\bar{y}$  are the amounts of x and y purchased at when the price of x is  $p_x$  then to make sure that  $(\bar{x}, \bar{y})$  can still just be purchased, income has to be adjusted as follows:

$$(1) \quad I' = I + (p'_x - p_x)\bar{x}$$

Thus if  $(\bar{x}, \bar{y})$  was purchased when the price of x is  $p_x$  and income is I,  $(\bar{x}, \bar{y})$  can also be just purchased when the price of x changes to  $p'_x$  and income is adjusted according to (1). This is because just enough extra income is given or taken away (depending on whether  $(p'_x - p_x)$  is positive or negative) so that  $(\bar{x}, \bar{y})$  can still just be purchased. To check this note that

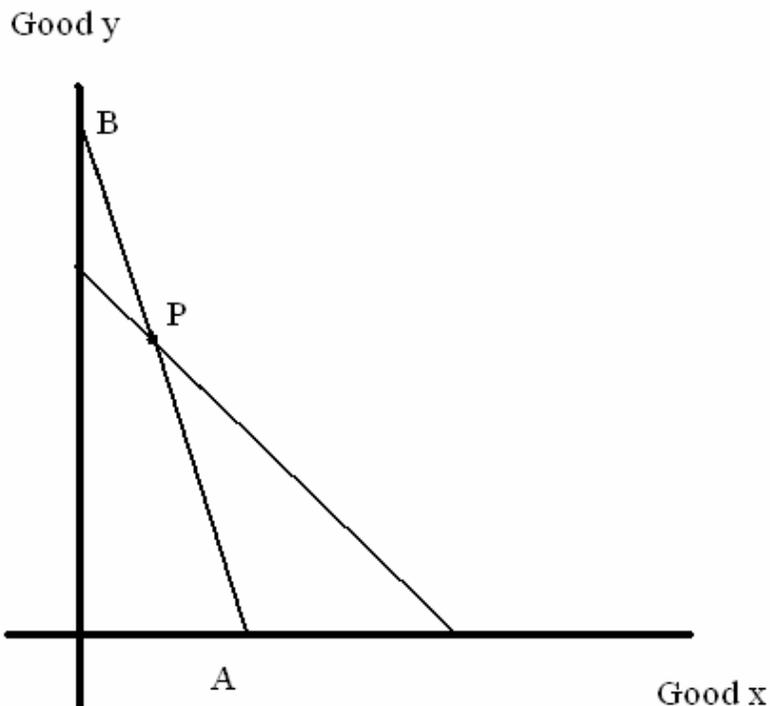
$$p_x\bar{x} + p_y\bar{y} = I$$

Substituting the left hand side of this expression into (1) and simplifying indicates that

$$p'_x \bar{x} + p_y \bar{y} = I'$$

So that  $(\bar{x}, \bar{y})$  can just be purchased when the price of x is  $p'_x$ .

An income compensated change in the price of x is shown in the diagram below. P is the original bundle purchased when the price of x is  $p_x$ . The price of x rises to  $p'_x$  and income is increased so that the bundle P still lies on the new budget set. Given the original choice P, choice along the line segment AP is inconsistent with utility maximization. Stated equivalently, these choices violate the weak axiom of revealed preference. Choices along the segment BP are consistent with utility maximization.



If individuals are picking points at random along the line BA, then some of the choices will violate the weak axiom of revealed preference, that is, be inconsistent with utility maximization. If average choices are examined, however, the average compensated demand curve will be downward sloping if there are enough individuals. This is because average demand corresponds to

Cobb-Douglas utility functions, that is, average demand curves can be derived from utility maximization. Thus, average choices will be consistent with utility maximization.

### Simulations of Compensated Demand

Here we have simulated compensated changes in prices. We see that individual behavior is sometimes inconsistent with utility maximization, but average behavior is consistent.

|      | 100      | 75       | 150      |          |          |
|------|----------|----------|----------|----------|----------|
|      | 2        | 1        | 4        |          |          |
| 1    | 11.60483 | 6.766905 | 3.022528 | 4.837926 | -8.5823  |
| 2    | 3.809173 | 30.81047 | 14.95791 | -27.0013 | 11.14874 |
| 3    | 22.27731 | 15.14131 | 11.87584 | 7.136001 | -10.4015 |
| 4    | 7.388525 | 31.42472 | 16.49949 | -24.0362 | 9.110967 |
| 5    | 7.975241 | 24.24892 | 17.43604 | -16.2737 | 9.460803 |
| 6    | 20.45888 | 3.874993 | 11.01942 | 16.58389 | -9.43946 |
| 7    | 0.747444 | 20.23135 | 0.835671 | -19.4839 | 0.088226 |
| 8    | 4.58232  | 11.10652 | 14.96036 | -6.5242  | 10.37804 |
| 9    | 12.21937 | 33.81067 | 17.75734 | -21.5913 | 5.537966 |
| 10   | 17.92704 | 7.576796 | 18.03264 | 10.35025 | 0.105595 |
| N=10 | 10.89901 | 18.49926 | 12.63972 |          |          |
| N=50 | 6.218405 | 8.215539 | 4.702093 |          |          |

Choices that violate the weak axiom of revealed preference are marked in red. Also, when the sample is small (N=10), average choices also violate the weak axiom of revealed preference.

When the sample is larger, however, average choice is consistent with a downward sloping compensated demand curve, as one would expect.

## **Conclusion**

The approach is somewhat abstract but is useful for illustrating some important points about the robustness of downward sloping demand, compensated or uncompensated, when the number of participants in the market is large. In our examples, 50 individuals are more than sufficient to accomplish this. This was of course the intent of Becker's 1962 article – to argue that downward sloping demand does not depend on the economist's conventional notions of rationality. This is clearly the most important point to make here.

There are some additional points that emerge from this kind of example. Most important is illustrating how economists actually go about the practice of economics. Economists are by and large interested in market behavior, that is to say average behavior. The rationality hypothesis is very useful in making predictions about average behavior. It is also useful in interpreting and explaining average behavior. Individual violations of rationality are not particularly irksome to economists.

References:

Becker, Gary (1962), "Irrational Behavior and Economic Theory," *Journal of Political Economy*, Volume 70, No. 1 (February 1962), pages 1-13.